# Study of connecting point with three-dimensions and four-dimensions by pictorial art 

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\text { Part } 1 \sim 10
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## PREFACE

## Study of connecting point with three-dimensions and four-dimensions by pictorial art Part 1~10


#### Abstract

We hope that through reading these articles Part 1~10 from the situation of topology, you could more easily understand our book " Unification of the Electromagnetic Willpower and Gravity of Four-Power of the Space: Proof by (1) -(4) for Digest"


In our K.I. RESEARCH INSTITUTE we study IKOSOLID SCIENCE by depending on art and scientific fusion for the purpose of direct contribution to evolution and the improvement of the earth and the human.

The Mr. and Ms. us was aimed for contribution to the world peace by the art and married. IKOSOLID was totally born like our child on May 22, 2001. Art name of IKOSOLID is REAL CUBE. [ Cf. first article]

IKOSOLID is the name that registered a trademark with the J apan Patent Office .
As a modern art art object REAL CUBE" our art activity did not have an other idea at first. Because there were art characteristics and the scientific nature in REAL CUBE, we described " Study of connecting point with threedimensions and the four dimensions by pictorial art" as a message article to the science from art.

If the first article is read, it is clear to be a picture article. It becomes the place of important expression of the painter that a picture is a two-dimensional body in the three-dimensional world. When a painter draws a picture as one piece of plane two-dimensional figure, the sight of the painter has an original image and a virtual image. In the three-dimensional world, as for the existing thing, there are an original image and a virtual image by all means for one pair of the mirror sides, and only an original image is to exist. However, in labelling the existence of the virtual image to be reflected in the mirror for a painter as the illusion that does not have the $m$ mass, we cannot easily understand so, because the thought that there is only an original image in being reflected in the mirror by all means is funny. In this three-dimensional world, it is common sense to label a virtual image to be reflected in the mirror as "virtual image = illusion" from the beginning.

Please think about why an original image is accompanied by a virtual image in a mirror side pair by all means without a mirror whether there is not the virtual image. Don't you think that a mirror expresses that anything called a virtual image exists? In other words for some reason the three-dimensional world original image was supported by the mirror side pair called a virtual image it was and thought that there was not only an original image independently.
Then it was possible for an image of "REAL PICTURE $=1 / n$ squares" that collected the anti-three dimensions world that an original image chased a virtual image at all opposite to the three dimensions world that a virtual image chased an original image as an original image and a virtual image as one body thing, and ran a race, and ran a race all together.[ Cf. first article \& second article]

To be next, we made the plane MAGIC SQUARED PICTURE which has REAL PICTUREs on the surface. REAL PICTURE could continue on the side and continue lengthwise and continue diagonally, in the shape of a mirror side pair [ in the shape of symmetry] on the surface. However, the plane MAGIC SQUARED PICTURE intercepts the endless circulation in the mirror side pair of REAL PICTUREs in the outer fence side.

On IKOSOLID which is the infinite MAGIC SQUARED PICTURE of the solid, REAL PICTURE can get endless circulation in the shape of a mirror side pair [ in the shape of symmetry]. In other words a limited plane MAGIC SQUARED PICTURE became the infinite MAGIC SQUARED PICTURE of the solid, that is the first article. [Detailed reference: The 3rd, 4th, 5th or 6th, 8th and tenth articles The explanation about multilayered two-dimensional figures of space ]
This is the study of connecting point with three -dimensions and the four dimensions by 720 degrees phase turn of space ( 360 degrees order turn) and the anti-universe ( 360 degrees backlashing). [Detailed reference: The eighth article]

Article 1-10 are articles of the IKOSOLID SCIENCE of the plus SEPTIMALNOTATION of $1: 6$ mainly; and of the IKOSOLID SCIENCE become basic.

Alternatively, there are about 40 articles of the IKOSOLID SCIENCE of the minus SEPTIMALNOTATION of 1 : minus 8.

The IKOSOLID SCIENCE has + SEPTIMALNOTATION of $1: 6$ and - SEPTIMALNOTATION of 1 : - 8 . It is a message from art. The IKOSOLID SCIENCE integrates "gravity" with "electromagnetic willpower " of four-power of the space and operates by "frequency".

We hope that in mathematics (topology), the IKOSOLID SCIENCE is understood, and the purpose of direct contribution to evolution and the improvement of the earth and the human is achieved.

We attach these $1 \sim 10$ articles to help with understanding of " Unification of the Electromagnetic Willpower and Gravity of Four-Power of the Space: Proof by (1)~(14) for Digest". We think it not to be it in time of the reimportation.

We are happy if you read these articles from the viewpoint of "topology" of the mathematics system in particular.

## Conventional progress to the understanding of the IKOSOLID SCIENCE

Serving as the advertising of the practical use products (IKOS -1 - HIF IKOC -8 - HIF) of + SEPTIMALNOTATION of $1: 6$ announced with "CEATEC JAPAN 2004" in Makuhari of October, 2004, we explained understanding of the IKOSOLID SCIENCE in those days to about 500 representatives of electronic industrial companies and about 500 professors and associate professors of physics or electronics of the universities. Besides representative alone of the major electronic company, it was a result to have an answer from nobody. We could not have them understand it even if they knew our experimental phenomenon to surpass super-conductivity by the IKOSOLID SCIENCE at normal temperature as science.

Before the practical use product announcement from 2003 to 2004 , the IKOSOLID SCIENCE called CEO of Hitachi, Ltd., the president of Sharp, the president of IBM JAPAN, a vice-president of Sony, Manager of fundamental research laboratory of Hitachi, Ltd . became interested. However, researchers of them were not able to understand it .In addition, there was attention of the CEO of American IBM and took inspection (as quantum computer development technology) of three months, but it finishes not being understood by researchers. It was not recognized even if we showed proof. The reason is a textbook had priority over the fact at hand.

Therefore we thought how we could be understood, over and over again. When we send the second article "Two questions to mathematicians from art " to ten Nobel prize winners residing in the U.S.A. by an e-mail, replies came from four people and it was their answer that no one understood us . Soon one "I do not understand it, but think that this is topology". We had the significant answer that a mathematician of the topology might
understand. We looked for mathematicians of the topology with hope from the words of the Nobel prize winner to have possibilities for mathematicians of the topology to understand us. Some topology has a big department, and there are algebraic topology, differential calculus topology, low dimension topology, geometric topology, here, we approach it led by a mathematician of geometric topology. Including the meaning, we dispatch the tenth article from the first article.

We understood that the factor of the IKOSOLID experiment to finally surpass direct current super-conductivity at normal temperature was in the outbreak effect of the positron that did not do pair annihilation with an electron. There it is in the first article, but the relations of "electronic (a particle) and the positron" (an antiparticle) resemble relations with "an original image and the virtual image" to be reflected in the mirror closely.
The positron was an illusion rather than the virtual image of the mirror till Mr. Paul Adrien Maurice Dirac proposed electronic antiparticle existence, and Mr. Carl David Anderson discovered it.

As for us, a positron occurs by the method that is totally different from the electron of the pair creation that a high energy accelerator of the mainstream depends on worldwide and the method called the positron outbreak now .

It is a method to generate positron outbreak by the crystal structure of IKOSOLID.
The positron outbreak that does not usually occur between the sky and the electron in the conductor (an electric wire) and pair annihilation of the positron that should be an anti-electron by the crystal structure of IKOSOLID which is different from the electron and the positron of the pair creation that a high energy accelerator of the mainstream depends on worldwide and the method called the positron outbreak at all is thought that it may be said with the real image of the virtual image by the relations of "an original image and the virtual image" that seems to be a mirror now.
In other words it is the method of the real image of the virtual image (a positron) by the relations of "an original image (an electron) and the virtual image" (a positron) to be reflected in the mirror.

The greatest characteristic of this method does not affect pair annihilation of an electron and the positron.

And this positron occurs in the normal space where IKOSOLID was related to. In addition, this positron occurs in a conductor (an electric wire) which we connected to IKOSOLID. It is the IKOSOLID SCIENCE that we
can give a threedimensional world original image a change by making a virtual image to be reflected in the mirror a real image.

It is a splendid characteristic of the topology that such a phenomenon is really generated.

Therefore we are convinced that understanding is deepened in mathematics (topology) by all means now.

The IKOSOLID SCIENCE is understood in mathematics, and we think that it is IKOSOLID as the modern tip to become the origin module of the advanced scientific breakthrough.

It is present at the public scientific experiment that the European and American scientist by the art exhibition surpasses direct current super-conductivity at normal temperature and understands favorably till now. However, they think that it is understood as the IKOSOLID SCIENCE by all means by a viewpoint of the topology now, but, many times till now we hoped that IKOSOLID SCIENCE was sent from J apan to the world. We think it not to be it in time of the reimportation. We have such acute thought.

IKOSOLID
It lights up the point of the modern science in mathematics" topology".
It becomes the base of the world economy.
It becomes the key that plays an important role reflecting the three-dimensional world.

J uly 11, 2010
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# Study of connecting point with three-dimensions and four-dimensions by pictorial art 

## Part 1

June 2001

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## Introduction

The aim of this thesis mainly consisting of picture images is as follows.
We think the most superb gift of painters is the space perception. The theme of this thesis mainly consisting of picture images is to express the point of connection between three-dimensions and four-dimensions and the reality of the four-dimensional space, by exploiting this space perception, by painting pictures and by making three-dimensional models.

We think the painters' gift for perceiving space is a second to none ability of direct recognition among human abilities in various fields, because many of the methods to express the results of today's scientific researches based on the unique abilities of direct and intuitional space perception of many humble and famous successive painters. Among them, there are several ways of expression, whose establishment was indispensable to today's scientific development.

The first contribution is the establishment of the perspective. We understand Renascence painters' activities enabled it, which is the main source of scientific three-dimensional space expression in two-dimensional world. People who have seen the ceiling wall paintings of Michelangelo: the Creation, the Last Judgment (Great Assize), realize his unbelievable space perception without fail. In Frescos, painters have to paint within specified time before the plaster dries. So to paint the whole pictures on all the ceiling alone shows Michelangelo's superhuman space perception.

It is well known that painters such as Leonardo da Vinci and Michelangelo were famous not only for being painters but also for being the first class scientists and architects. Also, in successive painters, you can see scientific features just like the ones you find in Leonardo da Vinci.

Painters' other contributions to scientific world are: discovery of positive and negative of photography (French painters), research and verification of the three primary colors = primary colors of lights and primary colors of painting (impressionists and post-impressionists), direct volume perception of three-dimensional space $=$ three-dimensional expression basis $=$ all three-dimensional space can be expressed by
cylinders, spheres, and cones(Cezanne, partly by Pablo Picasso).
Further more, Cubists such as Pablo Picasso challenged to express four-dimensional space, but it is not achieved up to the present. The painters such as Pablo Picasso managed to reach the Origami Theory, but their expression style was strictly limited within the original two-dimensional original pictures, so they came to the standstill in four-dimensional space expression, We think.

The themes of our thesis are "Study from Three-dimensional Space to Four-dimensional Space by pictorial art" and "Three-dimensional Expression of Four-dimensional Space". Through a painter's viewpoint, we aim to complete Picasso-and-other painters' study of four-dimensional space expression.

## 1. Roses of Iguazu

(The original)
$200191 \mathrm{~cm} \times 91 \mathrm{~cm}$

Picture drawn in Chinese Ink
(on Japanese Paper)
Koei Endo


## 2. Symmetrical Pictures (Symmetrical Images)

created by taking photos


While the original picture exists two-dimensionally, Symmetrical Picture of its true image and its virtual image exists three-dimensionally. In three-dimensional space, the situation in which Symmetrical Pictures rotating at 360 degrees with interface of true picture and virtual picture as the axis is considered to be Symmetrical Picture.

While the true image and the virtual image always exist as a pair, our consciousness concentrates on the true image. So, usually the virtual image is not perceived, but appears on symmetrical catalysts such as water or a mirror. Various recording media and recording devices have been invented or discovered through the characteristic of this virtual image.


Symmetrical Pictures (Symmetrical Images) have two horizontal types and two vertical types. We can count two types because 2 horizontal types and 2 vertical types are the same respectively if we rotate two of them at 90 degree, but the difference between verticality and horizontality is important for the process of this thesis because symmetry is "painters' space perception $=$ symmetry (together with rotation)".

The characteristics of Symmetrical Pictures (Symmetrical Picture Images) are:

## o-1 same on both side

o-2 Symmetrical Picture has the characteristics of a "true image", so, by making a virtual image of the Symmetrical Picture symmetrically, a new Real Image Picture (real image ) depicted in Chapter 3 will be created.

## 3. Real Image Pictures and real images ( 8 basic forms of Magic Squared Pictures)

Symmetrical Picture with a true image and a virtual image from one original picture appears by photo-taking (Chapter 2)(on previous page). There are " 4 kinds of Symmetrical Pictures" according to the symmetrical combination of pictures.
In this chapter, new 8 kinds of picture images appear by the symmetrical combination of 4 kinds of Symmetrical Pictures. These new picture images are called "Real Image Pictures". The characteristics of this Real Image Picture are:
"same on both sides"
"neither true image nor virtual image $=$ real image"
"image with the elements of infinitv.(elements of infinite expansion)"

o-1 horizontal

(1)

0-2 horizontal

o-3 horizontal

--4 horizontal


Just like Symmetrical Images of chapter 2, these 8 forms of Real Image Pictures are rotating at super high speed at 360 degrees, from the true image to the virtual image (or from the virtual image to the true image). Among these 8 kinds, the upper halves of $0-1$ horizontal $0-2$ horizontal $0-3$ horizontal $0-4$ horizontal are rotating at 360 degrees horizontally, and the lower halves of them are rotating at 360 degrees horizontally but reversely to the upper half rotation. Such condition is called "real image". This 360 degree positive rotation and 360 degree negative rotation are considered 720 degree rotation as a whole. We refrain from deciding whether this 720 degree rotation has some four-dimensional space elements or not, here.


0-4 vertical


Similarly, in case of o-1 vertical o-2 vertical o-3 vertical o-4 vertical, Symmetrical Picture (the right half of Real Image Picture) and Symmetrical Picture (the left half of Real Image Picture), are rotating at 360 degree each vertically in opposite directions just like Chapter 2). Such condition is called "real image". These 360 degree positive rotation and 360 degree negative rotation are considered 720 degree rotation as a whole. We refrain from deciding whether this 720 degree rotation has four-dimensional space elements or not, here.

The difference between a Real Image Picture and a real image is the difference between a plane picture and a solid, just like the difference between Symmetrical Picture and Symmetrical Image. We refrain from deciding whether these Real Image Pictures and real images have some three-dimensional or four-dimensional space characteristics.

From now on, the study of the connecting point with lower-dimensions and higher-dimensions by pictorial art begins, because we prefer to advance further without confirming everything. Further development of picture images suggests that they are just like solving mathematical and physical problems. Usually these problems of spaces are to be agued and assumed mathematically, but this thesis has the characteristics to solve such space problems by the images of pictures and has possibility to verify the space which denies mathematical analysis, by picture images, because these images have the means of clear symmetrical expressions of "true image", "virtual image", and "symmetry". Mathematicians and physicists are not supposed to have these means of symmetrical expressions, which we think is worth appreciating, even by this point only.

We suppose that quite a lot of Thomas Edison's inventions were done without mathematical and physical demonstrations, and that most of the inventions' foundations are applications of the three-dimensional characteristics such as "true image", "virtual image", and "symmetry". Such inventions are: plus and minus of electric currents, record and playback of sounds, wireless, invention of movies, illumination by resistance and flow of electric currents, and telephones.

## 4. Magic Squared Pictures (Magic Squared Images) $=3 \mathrm{D}$ picture images

We can say that Magic Squared Picture and Magic Squared Image in this thesis are so-called "3D picture image".
However, these are not created according to the 3D picture image theory but created by the symmetrical rule (with rotation movement). So they have factors to advance from 3D picture images to 4 D picture images.
"Meaning of Magic Squares here in this theory" is similar to the one of usual magic squares but rather may be called continuouslly expansional squares exactly. But in our mind, we have the magic square rule (vertically, horizontally, and diagonally) cotinuous development of the Real Image Picture and the real image in symmetrical pairs) originally, so we take the risk to use this word: "magic square", here. These Magic Squared Picture and Magic Squared Image in Chapter 4 and IKOSOLID in Chapter5 are created by the unique rules of "magic square". in our consciousness and throught


Figure A: Magic Squared Picture (Magic Squared Image) $=3 \mathrm{D}$ picture image


Here, this Magic Squared Picture (Magic Squared Image) has the characteristics of "being same on both sides", "being neither true image nor virtual image: being real image", and "being the image with the element of infinity (element of infinite expansion)". This infinite expansion's image is usually that of the universe expanding limitlessly, but the idea of this is endless expansion of finiteness, so long a way to go before reaching infiniteness.
So the Magic Squared Picture (Magic Squared Image) is finite and three-dimensional. That leads us to say that four-dimensions cannot be expressed on a two-dimensional plane surface. That is why Pablo Picasso could not express four-dimensional objects on his canvas \{(two-dimensional plane surface) and remained within the Origami theory (this 3D picture image can be said to be the development of three-dimensional origami (folded paper)\}.
These Magic Squared Pictures are the succession of Real Image Pictures (refer to Chapter 3.) Also, these Magic Squared Images are succession of real images in Chapter 3. (The Real Image Pictures changed three-dimensionally by horizontally positive and negative symmetry rotations and by vertically positive and negative symmetry rotations are called real images).


Figure A': Magic Squared Picture (Magic Squared Image) = 3D picture image


Two types of figures (Figure A and Figure A') can be made of these Magic Squared Pictures and Magic Squared Images. These are horizontal and vertical compositions. The horizontal composition includes all 4 horizontal types and the vertical composition includes all 4 vertical types.
In the following Chapter 5, on IKOSOLID surface and inside walls, Magic Squared Pictures and Magic Squared Images (A, A') are made into one. Further more, on its inside and outside walls, Magic Squared Pictures and Magic Squared Images can become infinite expansion condition completely endlessly. By changing the magic square's size of minimum basic constituent pictures or by changing the size of an IKOSOLID, the magic square can become smaller or bigger endlessly and the applications of them are limitless. We come all the way to "four-dimensional expression" by this IKOSOLID.

From an original picture to 4D picture images

5. IKOSOLID...4D picture
image...four-dimensional element a horizontal kind of Magic Squared Picture (Magic Squared Image) and a vertical kind of Magic Squared Picture (Magic Squared Image) are included.
"same on both sides", "neither true image nor virtual image", "with the element of infinity "...Magic Squared Pictures (Magic Squared Images) are put on the walls of a IKOSOLID in endless circulating
condition...
The inside and outside of an IKOSOLID is filled with four-dimensional elements...4D picture image.

## 5. IKOSOLID (IKOSOLID: trisoctahedron as a polyhedron= benzene ring solid as an image)

Magic Squared Pictures and Magic Squared Images (both their horizontal pictures and vertical pictures) were able to be put on the inside and outside walls of an IKOSOLID in the endless condition with the movement to expand their four sides infinitely. All the folds to make faces to compose this polyhedron are done according to the symmetrical rule, which means no folds without ones on sides and diagonal lines. This is the fundamental rule consistent (from the very beginning of this thesis) to create picture images. So, in creating this polyhedron, the same symmetrical rule is to be applied.

When the existence of IKOSOLID is once realized, we can see the full picture is done according to the symmetrical element. So, when we arrange another same IKOSOLID next to it, we can find that the two IKOSOLID's are in a symmetrical relation.

If we only aim to put Real Image Pictures (real images) on all the walls endlessly, a trigonal pentahedron just like pieces of cheese cake is possible, but this polyhedron is far from the symmetrical rule. It is difficult to create a polyhedron with folds only on sides and diagonal lines. In addition to that, we aim to create the polyhedron worth naming benzene ring (chemical) solid, and we now have this IKOSOLID.

It is the existence infinite in size, too. It has the element to realize the endless space of the universe, too. Its inside and outside walls have no right side and wrong side and is the endless magic squares with all 8 Real Image Pictures (4 horizontal types and 4 vertical types).

We proclaim the creation of four-dimensional space. Accurately, it is the creation of IKOSOLID existing in three-dimensional space, with four-dimensional elements. It is the space we can get inside.

Conclusion of "Study of connecting point with three-dimensions and four-dimensions by pictorial art Part 1"

For painters including Picasso, we can always see and feel the back of the objects together with space perception, even while painting in three-dimensions. We can say long-time cherished wish to objectively realize four-dimensional perception in three-dimensions materialized at last.

However, it does not just express four-dimensions in three-dimensions. Four-dimensional elements are to express in three-dimensional space. It is exactly the same as to express three-dimensional elements on the two-dimensional plane surface using perspective. The means and rules to express four-dimensional elements in three-dimensions, equivalent to this perspective are "symmetry (together with rotation movement)" and "endless circulation". These are the conclusion of this thesis: "Study of connecting point with three-dimensions and four-dimensions by pictorial art Part 1 "

## 6. Characteristics of IKOSOLID

In this Chapter 6, we would like to examine the characteristics of the IKOSOLID we actually created and to set up hypotheses on the movement of an IKOSOLID in painters' space perception, applying the symmetrical (with rotation movement) rule and the endless circulation rule. We have a motivation in ourselves to grasp five-dimensional elements to elevate to five-dimensions by using these hypotheses.

If you rotate an IKOSOLID centering the vertical axis, just like Figure "benzene ring solid" (originally, we are free to choose any angle of the axis because it is infinite space), a beautiful benzene ring solid appears on the outside edge of the rotating IKOSOLID. Also, if you connect each two points at the end of left and right concaves with lines, a benzene ring solid is produced.


Figure "benzene ring solid"


## o-1 Discovery and creation of IKOSOLID

Originally, we, I and my wife, started this IKOSOLID study for the purpose of making a solid with walls to put Magic Squared Pictures (Magic Squared Images) = 3D picture images endlessly for our three-dimensional picture gallery. As my inside space picture image, the solid has already clearly created, but it was a six-faced regular cube like a die, but I knew the shape would change when once it was created in three-dimensional space. I thought and thought to come to a dead end, so I decided to sleep on it.

My wife Ikuyo heard my word, "tetrapod" in my sleep, she got a hint from it, and created a magic squared solid within 15 minutes. (Figure o-1) I showed the picture of it to one of my friends on May 20, 2001.


Figureo-1 the first successful creation It was possible to put 3D picture images endlessly on its walls, but the symmetrical rule was not applicable Created by Koei and Ikuyo Endo on May 15, 2001

Figureo-2 Magic squared solid It was possible to put 3D picture images endlessly on the walls, but the symmetrical rule was not applicable. Created by Koei and Ikuyo Endo on May 21, 2001


Figureo-3 IKOSOLID \#1 discovered and created by Koei and Ikuyo Endo on May 22, 2001

He said "Is it possible to make a benzene ring solid?" Then, I replied "Yes."
I came back home, and I was sure of making one. My wife, at first, made a solid like pieces of cheese cake (Figure $0-2$ ) but symmetrical rule didn't apply to it. Next, she made KISOLID \#1 (Figureo-3). It was made of 1 cm -squared paper. Once it was made, we could say it was very easy, but it was an unthinkable shaped solid. We put Magic Squared Pictures on the solid's surface, and they circulated endlessly


Figure 4 Outward appearance of an IKOSOLID
When horizontally rotated at 360 degrees, the most outward sides make Benzene ring solid
(Figure in p.14)

Figure 5 IKOSOLID from just above



Figure A IKOSOLIDs' upper and lower points interface


Figure B IKOSOLIDs' edge interface of oblique edge


Figure C IKOSOLIDs'
two bilateral points interface (simultaneously)
o-2 Edge interface and point interface of IKOSOLID's
We try to combine a pair of completed IKOSOLID's using the combination way of benzene rings: combinations of upper and lower points interface (Figure A), edge interface of oblique edge (Figure B), and two bilateral points interface (Figure C).
We understand that each one of the pair is self-rotating, approaching ultimately each other. And as for one pair's self-rotation, they are rotating positively and negatively at 360 degrees each, which means 720 degree rotation like $\infty$ (infinite mark). That is because at the time of combinations in Figure A, Figure B, and Figure C, the interfaces become symmetrical relations between combining points or combining edges. We notice clear logics because the symmetrical pair approaching ultimately each other is reversely symmetrical on upper and lower side( in each case of convex-concave, convex-convex, or concave-concave combination). In the case of Figure C (two bilateral points interface), each point meets simultaneously at the farthest distance from each other.(Figure C) This is the discovery we did in this combination experiment. These two bilateral point pairs are different in angles, so when convex and concave face each other reversely symmetrically, they combine one at a time, which is the proof that each IKOSOLID pair rotates at 720 degrees.
o-3 Displacement (separation), combination, and rotation of IKOSOLID's in the state of symmetry

Photos of our experiment on the three types of combinations of IKOSOLID's using models are shown in $0-2$ on page 17. In this section $0-3$, our assumption is that IKOSOLID should have positive rotation and negative rotation at 360 degree each (rotation at 720 degrees in infinite state $\infty$ ) when solids are in symmetric position on the interfaces as axes and points to be combined (that should be described that they approach ultimately each other). This theory is very similar to the principle of three-dimensional motor. +and $+/$-and- are separated and +and- are combined by magnetic force. We assume that it should be applicable to some force to displace or rotate IKOSOLID's.
In symmetric relationship of IKOSOLID's, between [+and+] and/or [-and-] would be [separation $=$ displacement] and [separation $=$ self-rotation]. Separation should be needed for displacement to combine each other. At the time when it is changed separation force to rotation force, combination force (ultimate approach) would be activated at the interfaces of+and- in IKOSOLID .
Figure A': IKOSOLID's, upper and lower point interface
Suppose that face of + should be flat and - would be dented. In the state of symmetry, and 720 degree positive rotation and negative rotation like $\infty$ (infinite mark) are provisional:


720 degree rotation like 8 (infinite mark) on the interface.


Figure B': IKOSOLID's, edge interface of oblique edge


Combination on the condition of +and-


Stop of rotation instantaneously while combination


720 degrees rotation like $\infty$ (infinite mark) on the oblique line


Separation $=$ Displacement $=$ Self-rotation in combination on the condition of + and +

Figure C': IKOSOLID's, two bilateral points interface (simultaneously)


Displacement $=$ Self-rotation in combination on the condition of + and + , -and-


They are rotating at 720 degrees like $\infty$ (infinite mark) at the left and right combination points.
o-4 Self-rotation of IKOSOLID's, and rotations of Symmetrical Pictures and Real Image Pictures (Symmetrical Images and real images)

As shown in Figure $\mathrm{A}^{\prime}, \mathrm{B}^{\prime}$, and $\mathrm{C}^{\prime}$ on p .18 , we displayed the hypotheses by photos when IKOSOLID's are combined in 3 different ways. These hypotheses are that in combination between [+and+]and/or[- and -], symmetrical relationship of IKOSOLID's edge interface and point interface would be separation= displacement= self-rotation at the time of combination In combination between [+and-], at combination on edge interface and point interface, IKOSOLID's would stop rotation instantaneously then start self-rotation at combination.

At these times, IKOSOLID's will have combinable or non-combinable characteristics, so the characteristics of IKOSOLID's are changeable depending on its rotation or its stop position.

As seen on the section of IKOSOLID while self-rotating, other combinations or separations can be thinkable except the combination at the time of instantaneous rotation stop. As it is three-dimensional and self-rotating, the combination at the back is also thinkable. When once we combine IKOSOLID models, we will be able to find the cue. Self rotation, stop (combination), and combination while self-rotation are also thinkable. We need more time to finish these ideas in our own way, so we stop writing for a while.

Next, several IKOSOLID samples of self-rotation and combination are displayed. As a premise, IKOSOLID self-rotation for each is 360 degrees and IKOSOLID's self-rotation as a pair is 720 degrees. For more IKOSOLID groups, we are sure the symmetrical rule is applicable to these two simple and multiple IKOSOLID's (one IKOSOLID and a pair of IKOSOLIDs). Just like the original picture became Symmetrical Picture (Image), Real Image Picture (Image), and Magic Squared Picture (Image) and put on the outside and inside walls of an IKOSOLID in the former half of this thesis, an IKOSOLID can become a real image, and be in Magic Squared Image condition, then it can become IKOSOLID (IKODOEITSCUBE) just the image we had originally. My space perception as a painter imagined a clear image of the solid when we created an IKOSOLID by the symmetrical rule. According to my image, my wife created a trisoctahedron i.e. IKOSOLID. As I am pressed for time, I cannot pursue it for a while. I imagine that three-dimensional combination of more than four IKOSOLID's will make the solid I want to create, which is "Cube of Idea" by Plato. A three-dimensional die which is the combination of IKOSOLID's, exists as prototype of the die in four-dimensions or in higher dimensions. We want to pursue this by combining IKOSOLID models in future.

In this thesis, attached IKOSOLID models are indispensable for the element of thesis expression. Also, we think three-dimensional models are prerequisite for the thesis on four-dimensions.


When Symmetrical Picture rotates at 360 degrees, it becomes Symmetrical Image (three-dimensional solid) (refer to p.2, p.3)


The upper half of Real Image Picture rotates at 360 degrees positively while the lower half of Real Image Picture rotates at 360 degrees negatively ( 720 degrees rotation), then a real image ( a three-dimensional solid with four-dimensional elements and characteristics of infinite continuation) appears. (refer to p.4, p.5)


Even one IKOSOLID can be a real image (four-dimensional solid) when (self-)rotating


While rotating, each one pair points combine one at a time with time lag (in case of convex-concave connection). While stop rotating, one pair points is combined simultaneously. (in case of convex and convex combination, and concave and concave combination)

When a pair of IKOSOLID's (as one unit) rotates at 720 degrees in $\infty$ (infinite mark) condition, the pair of IKOSOLID's becomes a real image
(four-dimensional solid)

When four IKOSOLID's are put together in symmetry to each other, they become a real image (four-dimensional solid). When two stages and two lines of IKOSOLID's rotate at 720 degrees in $\infty$ (infinite mark) condition, they also become a real image (four dimensional solid). If these symmetrical two stages are considered a real image, this real image can be continuously and infinitely developed as a Magic Squared Image. Then after the full self-rotation, combination with the same numbers of the front rotating real images at the back of Magic Squared Image as the depth is assumed possible. Then the solid appeared is to be "Cube of Idea by Plato". (This Cube of Idea by Plato has been in our inside space from the beginning, and later appeared as IKOSOLID's = "IKODOEITSCUBE".)

# "Study of connecting point with three-dimensions and four-dimensions by pictorial art" 

## Part 2

## Two questions for mathematicians from art

1. Four- dimensional geometric problem (IKOSOLID as a catalyst)
2. Four- dimensional integral problem (IKOLSOLID as a catalyst)

May 2002

Koei Endo
Ikuyo Endo

## Chapter 1. Two questions for mathematicians from art

1. Four-dimensional geometric problem (IKOSOLID as a catalyst)
2. Four-dimensional integral problem (IKOSOLID as a catalyst)

These are the questions to have "mathematical solutions" for the process of dimensional penetration using an IKOSOLID used practically in arts as a catalyst.

IK OSOLID: Created by Koei Endo and Ikuyo Endo.
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## Section 1

Four-dimensional geometric problem (IKOSOLID as a catalyst)

We are to prove that the definition of "IKOSOLID should be a projective figure of a regular octahedron by Plato", proposing "four-dimensional geometric problem (using IKOSOLID as a catalyst).

We request mathematicians and scientists to investigate it as a four-dimensional geometric problem and to find a solution from mathematical point of view.
A. Two-dimensional development (like origami \{folded paper\}) of a regular octahedron by Plato as the base of a projective figure for an IKOSOLID (called hereafter).

Figure 1


Photol


## B. Regular octahedron by Plato

Figure 2


Photo 2

C. View of a regular octahedron by Plato from above

It is not only a projective but also a perspective figure in pictorial art. The angles are changed because 8 pieces of equilateral triangle consisting three-dimensionally of a regular octahedron by Plato are transformed to right-angled isosceles triangles.

Figure 3


Photo 3

D. Four-dimensional development of the projective figure of a regular octahedron by Plato.
It is a theory of cubism by Picasso. Based on the perspective, which is to draw something behind, it is developed like origami (folded paper). Painters generally apply multi-point perspective. The number of the points depends on each painter.
The reverse sides of the projective regular octahedron (perspective view from right above) are folded up.

| Figure | 4 | Photo 4 |
| :--- | :--- | :--- |$\quad$ (Four-dimensional

(View of the regular octahedron by Plato in the four-dimensional geometrical state from right above and right under at a time).


Figure 4-2 Side view view
(A general thinking way of folding the reverse sides to the horizontal position)


Photo 4-2 Three-dimensional side


Photo 4-3 (general thinking way)


* Figure 4 (Four-dimensional projection $[$ perspective)

Four-dimensional projective and perspective which are to see 8 faces on a regular octahedron by Plato simultaneously" can be revealed in the three-dimensional space (Photo 4) and is drawn with four-dimensional geometry (Figure 4).

By folding it like two-dimensional origami (folded paper), the five-dimensional existence can be produced three-dimensionally.
It is the same action to produce a regular octahedron by Plato in three-dimensional space (Figure 2, Photo 2) with two-dimensional origami (Figure 1, Photo 1), but dimensionally they are different. We can see similar relationship between two-dimensions and four-dimensions and the one between three-dimensions and five-dimensions. How can the five-dimensional existence be shown three-dimensionally? Perspective drawing is a key. To penetrate dimensionally is derived from "symmetric rule and endless rule by isosceles triangle and square". In four-dimensional geometry examined herein, plane object is always perspectively and projectively plane (Figure 3 and Photo 3). If it is transformed dimensionally, it has a nature to return to the figure, which is perspectively and projectively three-dimensional plane object (Figure 3 and Photo 3) by squashing from right above. The five-dimensional existence is "made plane by remaining in three-dimensional perspective and transforming dimensionally".
It has been told from the old time that a regular octahedron by Plato is the only sacred solid linking to four-dimensions. Our idea is that it is a significant "catalyst" to link three-dimensions with four-dimensions. A half of a regular octahedron by Plato is the same figure of well-known Pyramid. The deeper and deeper it is examined, it would be understood obviously that "IKOSOLID" is a catalyst to link three-dimensions with five-dimensions. It has been already applied in art practically. It is also possible to "apply it scientifically by four-dimensional integration". It would be "a study of plane finite magic square and dimensional infinite magic square by using a IKOSOLID as a catalyst".
E. To fold a piece of paper two-dimensionally like origami (which transforms four-dimensional geometry to two-dimensional one) with a regular octahedron by Plato developed four-dimensionally. One fold is reduced and the angle of fol ding is changed.

Figure 5


Photo 5


## F. A quarter of an IKOSOLID with four-dimensional origami.

One projective figure of a regular octahedron by Plato can be one piece of "IKOSOLID"

Proof: The definition, " a projective figure of a regular octahedron by Plato is a solid" can be interpreted that "one piece of projective figure of a regular octahedron by Plato is a quarter of "IKOSOLID." Assuming that one of isosceles triangles on a regular octahedron by Plato (Figure 4) would be al, 8 pieces (a8) are on it.
On a quarter of IKOSOLID, there are 2 pieces $(a 2 \times 2=a 4)$ of b1 ( $=a 2$ ) which is double area of a1, and 4 pieces of al ( $=a 4$ ) are left. Total is a8. Therefore, "a" of a regular octahedron by Plato is equivalent to "a" of an isosceles triangle on a quarter of IKOSOLID.

## Photo 6 (a quarter of IKOSOLID)



Photo 6-3 (a quarter of IKOSOLID)

Photo 6-2 (a quarter of IKOSOLID)


Photo 6-4 (a quarter of IKOSOLID)


* By squashing a quarter of IKOSOLID from right above, it is returned to a projective figure of a regular octahedron by Plato (Figure 3, Photo3). It shows that the nature of IKOSOLID is consistently "projective for a regular octahedron by Plato.
(Squashing IKOSOLID in Photo 6 from right above and making it flat by force, it is returned like Photo 3 (Figure 3). Precisely described, it will be another isosceles triangle (b) which area is double of one isosceles triangle (a) in Figure 3 and Photo 3.)

Photo 7 (Squashing it in Photo 6-3)


Figure 6 (figure of Photo 7)

Photo 3 (reference)


G. A half of an IKOSOLID. A half of an IKOSOLID can be made if two pieces of a quarter of an IKOSOLID are combined at each 2 side of the opening part. (It is not completely closed at the opening part. Two pieces of an IKOSOLID can be completely closed and combined in the form of cross intersection.)

Photo 8


Photo 8-2


## Photo 8-3



* By squashing a half of an IKOSOLID from right above, it is returned to the projective figure of a regular octahedron by Plato (Figure 3, Photo 3) as a quarter of an IKOSOLID does. It shows that the nature of an IKOSOLID is consistently the "projective figure" of a regular octahedron by Plato. It should be noted that the folding lines are the same as the lines in Figure 3 and Photo 3, but one opening line is made.

Photo 9-1
Opening line: c
Folding line: d

Photo 9-2


Back

## H. A finished IKOSOLID

Photo 10


Proof: "By combining 4 projective figures of a regular octahedron by Plato, one piece of an IKOSOLID is made." It can be proved that the definition of "an IKOSOLID is a projective figure of a regular octahedron by Plato" means "projective figure of a regular octahedron by Plato, 4: IKOSOLID 1". On the surface of an IKOSOLID, there are 24 pieces of two kinds of right-angled isosceles triangles. Assume that one piece of right-angled isosceles triangles in four-dimensional projective figure of a regular octahedron by Plato (Figure 4 and Photo 4) (all 8 pieces are visible at a glance by perspective. Recognition of four- dimensional space) should be al, and it shows that there are 8 pieces of al $=a 8$.

Right-angled isosceles triangles on IKOSOLID are 8 pieces $\{(a 2) * 8=a 16\}$ of b1 (a2) which is double area of a1 and 16 pieces of a1\{(a1)*16=a16\}are left. The total is a32. a8 (the right-angled isosceles triangle of a four-dimensional projective figure of a regular octahedron by Plato) multiplied by 4 equals a32. It shows that "one piece of IKOSOLID is made of 4 pieces of four dimensional projective figure of a regular octahedron by Plato."


#### Abstract

* The relationship between a regular octahedron by Plato and IKOSOLID has been proved with figures and photos from the artistic point of views. As a next step, mathematical and scientific SOLUTION to a "problem in a four-dimensional geometry" should be required. Nowadays we have been in labyrinth of mathematical CHAOS and QUANTUM MECHANICS in theoretical physics. This is proposed because we wish the universe in harmony to make a scientific contribution to the hope and peace of mankind.


* Photos of an IKOSOLID taken from the different angles are followed for reference. It is noticed that it is impossible to draw IKOSOLID by computer used at the design offices or metal factories. It should be drawn by high-quality CAD or by hand. It is clearly shown "how IKOSOLID exists" through the process to draw.

As a catalyst to link five-dimensions with three-dimensions, it is not only a unique solid created three-dimensionally, but also a three-dimensional model which exists five-dimensionally and has dimensional penetration elements. A fact that it has a spatial element two-, three-, four-, and fivedimensionally would cause impossibility to draw by CAD commonly used.

In short, by using origami (two-dimensional existence), three-dimensional existence can be shown. If a solid projected four-dimensionally (it is four-dimensions if the whole body of a solid is seen in perspective; it is four-dimensions if the 6 faces of a die can be seen cubically at a glance.) would be interpreted in four-dimensional plane geometry and folded like two-dimensional origami, it is possible to make it three-dimensionally= three-dimensional existence while a five-dimensional existence (five-dimensional element) is still remained. It is our theory that three-dimensional existence can input records in two-dimensions and five-dimensional existence can input records in four-dimensions. We draw three-dimensional space on a two-dimensional canvas in perspective. We have sensed from our experiences as painters that five-dimensional space should be also drawn in two-dimensions because we found out that two-dimensions is quite similar to four-dimensions and three-dimensions is to five-dimensions. The relationship between Plato's idea (existence) and its shadow is recognized in between five-dimensions (existence) and three-dimensions (shadow). The relationship between four-dimensions
(existence for recording dimension) and two-dimensions (shadow for recording dimension) is our space recognition. Painters have dealt with space in front of us practically, not with theories and mathematical principles. Few admires Paulo Uccello (13th century to 14 th century), a first painter of perspective drawing, as the greatest painter in the world. However some artists including our teacher Mr. Paul Ambille, one of the best painters, respect Uccello' s achievement. We believe that the achievement of Uccello, Cezanne, and Picasso should also have contributed scientifically. We hope our research would be a further study of Picasso's.

Photo 11 (from the different angles)


It was found that 1) IKOSOLID has $50 \%$ of + and $50 \%$ of - on the surface, and 2) relative faces are combined symmetrically however multiply and / or repeatedly same poles are combined (+ and +, - and -) when benzene combination is made.
Geo-science used to express benzene combination flatly. But it is found that it is repeated (several times is possible) on the back (inner part) of multiplecombination. IKOSOLID's capability as a catalyst to make it possible in the state of benzene solid is immeasurable.
As it is stated in Section 2, when an IKOSOLID consists of one pair of solids combined, with the movement of 720 degrees rotation or still, and if it is combined by odd number, one IKOSOLID in between can act as an common axis. In the multi-combination and / or repeated-combination, a pair by side also rotates at 720 degrees and / or stand still.
The nature of IKOSOLID was detailed in our first paper. Adding to it, it is amazing that two combination points on one IKOSOLID in the horizontal combination are out of perpendicular line, but definitely attached.
A gap at the points in the horizontal combination suggests "screw of rotation.

Fundamental nature of IKOSOLID would be "symmetry" and "infinity of endlessness".

We believe that IKOSOLID should be a fundamental existence to construct the universe. As it is much smaller than atoms and other elements on the earth, it would be accurately an original model of IKOSOLID showing three-dimensionally to represent IKOSOLID in five-dimensions. Therefore, it is "a catalyst to link three-dimensions with fivedimensions". It is significant that a catalyst can act on condition that the relationship between "two-dimensions" for us to record something in three-dimensions and "four-dimensions" for five-dimensional existences to record is established.
It has not been well aware that ability and contribution of "painters" had a great impact on the study of two-dimensions and four-dimensions.
This is intentionally composed to propose questions for all of mathematicians from art.

## Section 2.

Four-dimensional integral problem (IKOSOLID as a catalyst)
Original picture $\rightarrow$ Symmetric picture $\rightarrow$ Real picture $\rightarrow$ Magic-Squared Picture
$\rightarrow$ IKOSOLID
The relationship between them is demonstrated in three ways of pictorial art, point-numbers, and numbers.
A number of the real pictures is described as " n ". The number of real pictures in Magic Squared Picture and IKOSOLID is formulated:
$\mathrm{X}^{2} \times 16=\mathrm{n} \quad$ Magic Squared Picture (finite plane in the state of magic square)

Called as K. I. Theorem
$\mathrm{X}^{2} \times 16=\mathrm{n}^{\infty}$ IKOSOLID (infinite solid in the state of magic square) called as
K. I. -2 Theorem
( X is a vertical or horizontal number of real pictures in one piece of square on plane Magic Squared Picture divided into 16 pieces)
We request a mathematical proof to show an integral SOLUTION as a four-dimensional integral problem. As a pictorial art is reasonable, it is important to have a SOLUTION that points and numbers should be considered as four-dimensional figures like 720 degrees rotation and number in reverse (points), and lower number (points). The number of real pictures on a Magic Squared Picture can be calculated by " n ", but it is also indispensable to have a mathematical SOLUTION to see the reason why " n " is endless and infinite multiplication in IKOSOLID.

Processes from an original picture to IKOSOLID are shown in three ways A: pictorial art, B: point-numbers, C: numbers:

Original picture $\rightarrow$ Symmetric picture $\rightarrow$ Real picture $\rightarrow$ Magic Squared Picture $\rightarrow$ IKOSOLID
(The above regarding A: pictorial art described in our first paper is stated again for reference for mathematical SOLUTION to B: point-numbers and C : numbers. It seems to be in high level in mathematics while this integral expression is commonly used in pictorial art. It has been studied and practically used in pictorial that is to observe perspectively and to draw two-dimensionally as a picture. Painters discover their own abilities of multi-point perspective through their pictorial research. It is
high level recognition of space. This paper is our result from the research about our ability of space recognition.)
A. ORIGINAL PICTURE of pictorial art

B. ORIGINAL PICTURE of point-numbers

C. ORIGINAL PICTURE of numbers


Symmetrical Picture
It is that two pictures are positioned symmetrical on right and left side or on upper and lower side. It is a three-dimensional space that a true image and a vertical image of the original picture are paired with and rotate at 360 degrees on an interface as an axis. Almost all of science on the earth is related to the rule of 360 degrees rotation. There are two types of horizontal rotation and vertical rotation each.

NOTE：Marks 1
showing swirling direction are used for reference only as it is opposite direction in reverse．$\quad$ See $\rightarrow \leftarrow$ ．

A．SYMMETRY of pictorial art


O）$\uparrow$
B．SYMMETRY of point－numbers


C．SYMMETSRY of numbers

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## A. REAL PICTURE of pictorial art

o-1. 4 types of horizontal rotation, 720 degree rotation


## A. REAL PICTURE of pictorial art

o-2. 4 types of vertical rotation, 720 degree rotation

B. REAL PICTURE of point-numbers
o-1. 4 types of horizontal rotation, 720 degree rotation

$\rightleftarrows$

$\checkmark$
B. REAL PICTURE of point-numbers
o-2. 4 types of vertical rotation, 720 degree rotation

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C．REAL PICTURE of numbers
o－1． 4 types of horizontal rotation， 720 degree rotation

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| $\downarrow$ | 9 | $\varepsilon$ | 3 |  | $\vdots$ |
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e


G
C. REAL PICTURE of numbers
o-2. 4 types of vertical rotation, 720 degree rotation

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| $a_{1}$ | 1 | - | $\rightarrow$ | $\Omega$ | $\infty$ |
| + | $m$ | $\infty$ | $\infty$ | $m$ | $t$ |
| 7 | $\omega$ | $\infty$ | $\infty$ | $\omega$ | $f$ |
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| $c$ | 5 | 0 | $\infty$ | $x$ | $\omega$ |



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| $\infty$ | $n$ | $t$ | + | $m$ | $\infty$ |



A. Magic Squared Picture by pictorial art
o-1. 1 type of horizontal rotation, 720 degree rotation


| $\xrightarrow{+}$ | $\xrightarrow{+}$ | $\stackrel{+}{+}$ | $\xrightarrow{\rightarrow}$ |
| :---: | :---: | :---: | :---: |
| 4 | 4 | + | 4 |
| $\dagger$ | + | $\dagger$ | $\xrightarrow{+}$ |
| $\leftrightarrow$ | 4 | 4 | + |
| $\rightarrow$ | $\rightarrow$ | $\xrightarrow{+}$ | $\dagger$ |
| 4 | 4 | + | 4 |
| $\dagger$ | $\xrightarrow{+}$ | $\dagger$ | $\dagger$ |
| $\leftarrow$ | $\leftarrow$ | 4 | 4 |

$$
\begin{aligned}
& 1^{2} \times 16=\mathrm{n} \\
& 1^{2} \times 16=16
\end{aligned}
$$



Real picture 16
--2.1 type of vertical rotation , 720 degree rotation


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| $\uparrow \downarrow$ | $\uparrow$ | $\uparrow$ | $\pm$ |  | 41 |
| $\hat{f}^{+} \mid t$ | $\uparrow 1+$ | f | t |  |  |
| $\cdots$ | キ | 4 | $\pm$ |  |  |

$$
1^{2} \times 16=n
$$

$$
1^{2} \times 16=16
$$



Real picture 16

## B. Magic Squared Picture by point-numbers

--1. 1 type of horizontal rotation, 720 degree rotation

o-2. 1 type of vertical rotation, 720 degree rotation

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| :---: | :---: | :---: | :---: |
| + | 4 | + | 4 |
| $\rightarrow$ | + | $\dagger$ | $+$ |
| ↔ | 4 | + | 4 |
| $\rightarrow$ | + | $+$ | $\stackrel{+}{+}$ |
| 4 | 4 | + | 4 |
| $\xrightarrow{+}$ | $\stackrel{ }{+}$ | $\pm$ | $\xrightarrow{+}$ |
| + | T | 4 | 4 |

$1^{2} \times 16=\mathrm{n}$
$1^{2} \times 16=16$


$$
\begin{aligned}
& 1^{2} \times 16=n \\
& 1^{2} \times 16=16
\end{aligned}
$$



Real picture 16
－－1 1 type of horizontal rotation， 720 degree rotation

| 9 | 1 | 8 | 8 | 1 |  | ė | 9 | 1 | 8 | 8 |  |  | e | 9 | 1 | 8 |  |  | 1 | é | 9 | 1 | 8 | 8 | 1 | e |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | 5 | $\varepsilon$ | 3 | 2 |  | $\pm$ | L | 5 | $\varepsilon$ | 3 |  | 2 | 3 | L | 5 | $\varepsilon$ | 3 |  | 2 | $\pm$ | 4 | 5 | $\varepsilon$ | 3 | 2 | d |
| $\tau$ | 6 | $\rightarrow$ | ＋ | § | $\delta$ | 5 | $\tau$ | $\overline{6}$ | म | ＋ |  | $\bar{\delta}$ | J | $\tau$ | $\overline{6}$ | म |  |  | § | $\delta$ | $\tau$ | 6 | ＋ | ＋ | 亏े | 5 |
| 5 | 9 | ＋ | 4 | 9 | 9 | 2 | 5 | 9 | $+$ | 4 |  | 9 | 2 | 5 | 9 | ＋ |  |  | ？ | 2 | s | ？ | $+$ | 4 | 9 | 2 |
| F | ट | $\varepsilon$ | 3 | 5 |  | 7 | F | ट | $\varepsilon$ | 3 |  | 5 | 7 | F | 己 | $\varepsilon$ | 3 |  | 5 | 7 | f | ट | $\varepsilon$ | 3 | 5 | 7 |
| 2 | $t$ | 8 | 8 | 1 |  | 6 | a | 1 | 8 | 8 |  | 1 | 6 | $\stackrel{\rightharpoonup}{\square}$ | ＇ | 8 |  |  | 1 | 6 | a | $t$ | 8 | 8 | 1 | 6 |
| 9 | 1 | 8 | 8 | 1 | 1 | e | 9 | 1 | 8 | 8 |  | 1 | é | 9 | 1 | 8 |  |  | 1 | é | 9 | 1 | 8 | 8 | 1 | è |
| t | 5 | $\varepsilon$ | 3 | 2 |  | 5 | L | 5 | $\varepsilon$ | 3 |  | 2 | $\pm$ | L | 5 | $\varepsilon$ |  |  | 2 | t | t | 5 | $\varepsilon$ | 3 | 2 | 3 |
| $\tau$ | 6 | 力 | ＋ | $\delta$ |  | 5 | $\tau$ | 6 | ＋ | ＋ |  | ¢ | 5 | $\tau$ | \％ | サ |  |  | $\delta$ | 5 | $\tau$ | 6 | خ | ＋ | $\delta$ | 5 |
| $s$ | $\rho$ | 4 | 4 | ？ | 9 | 2 | $s$ | 9 | $+$ | 4 |  | $?$ | 2 | s | ？ | ＋ |  |  | ？ | 2 | \＆ | ¢ | ＋ | 4 | 9 | 2 |
| f | を | $\varepsilon$ | 3 | 5 |  | 7 | P | 己 | $\varepsilon$ | 3 |  | 5 | 7 | 7 | E | $\varepsilon$ | 3 |  | 5 | 7 | F | ב | $\varepsilon$ | 3 | 5 | 7 |
| a | $t$ | 8 | 8 | 1 |  | 6 | a | $t$ | 8 | 8 |  | 1 | 6 | ¢ | 1 | 8 |  |  | 1 | 6 | a | 1 | 8 | 8 | 1 | 6 |
| 9 | 1 | 8 | 8 | 1 |  | é | ¢ | 1 | 8 | 8 |  | 1 | é | 9 | 1 | 8 |  |  | 1 | é | 9 | 1 | 8 | 8 | 1 | ė |
| t | 5 | $\varepsilon$ | 3 | 2 |  | \＄ | 4 | 5 | $\varepsilon$ | 3 |  | 5 | d | L | 5 | $\varepsilon$ | 3 |  | 2 | t | L | $\leq$ | $\varepsilon$ | 3 | 2 | ¢ |
| $\tau$ | $\overline{6}$ | 万 | ＋ | ¢ | 5 | 5 | $\tau$ | $\overline{6}$ | サ | ＋ |  | $\delta$ | 5 | 乙 | 6 | 力 |  |  | $\delta$ | 5 | $\tau$ | 6 | $\dagger$ | ＋ | $\delta$ | 5 |
| $s$ | ？ | 4 | 4 | 9 | 9 | 2 | s | 9 | 4 | 4 |  | 9 | 2 | $\bigcirc$ | ¢ | ＋ |  |  | 9 | 2 | 5 | ？ | ＋ | 4 | 9 | 2 |
| f | e | $\varepsilon$ | 3 | 5 | 5 | 7 | 7 | 己 | $\varepsilon$ | 3 |  | 5 | 7 | F | 己 | $\varepsilon$ | 3 |  | 5 | 7 | $f$ | ट | $\varepsilon$ | 3 | 5 | 7 |
| 2 | 1 | 8 | 8 | 1 | 1 | 6 | a | 1 | 8 | 8 |  | 1 | 6 | a | $t$ | 8 |  |  | 1 | 6 | a | 1 | 8 | 8 | 1 | 6 |
| ¢ | 1 | 8 | 8 | 1 |  | ė | 9 | 1 | 8 | 8 |  | 1 | é | 9 | 1 | 8 | 8 |  | 1 | é | 5 | 1 | 8 | 8 | 1 | e |
| L | 5 | $\varepsilon$ | 3 | 2 |  | \＄ | $t$ | 5 | $\varepsilon$ | 3 |  | 2 | $\pm$ | $t$ | 5 | $\varepsilon$ | 3 |  | 2 | $\pm$ | L | 5 | $\varepsilon$ | 3 | 2 | $\pm$ |
| $\tau$ | 6 | \＃ | ＋ | हो |  | 5 | $\tau$ | 6 | 力 | ＋ |  | $\delta$ | $\delta$ | $\tau$ | 6 | ＋ | t |  | ठ | 5 | 7 | 6 | H | ＋ | $\delta$ | 5 |
| $\checkmark$ | 9 | 4 | 4 | 2 |  | 2 | $\Omega$ | ¢ | 4 | 4 |  | $?$ | 2 | 5 | 9 | ＋ | 4 |  | $?$ | 2 | $\Omega$ | 9 | 4 | 4 | 9 | 2 |
| F | e | $\varepsilon$ | 3 | 5 |  | 7 | 7 | 己 | $\varepsilon$ | 3 |  | 5 | 7 | 7 | 己 | $\varepsilon$ | 3 |  | 5 | 7 | F | 己 | $\varepsilon$ | 3 | 5 | 7 |
| 3 | 1 | 8 | 8 | 1 |  | 6 | 2 | $t$ | 8 | 8 |  | 1 | 6 | a | 1 | 8 | 8 |  | 1 | 6 | a | $t$ | 8 | 8 | 1 | 6 |


$1^{2} \times 16=\mathrm{n}$
$1^{2} \times 16=16$

| 9 | 1 | 8 | 8 | 1 | é | Real picture 16 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $t$ | 5 | $\varepsilon$ | 3 | 2 | 3 |  |
| $\tau$ | 6 | 力 | ＋ | $\delta$ | 5 |  |
| $\bigcirc$ | $\rho$ | 4 | 4 | 9 | 2 |  |
| $\bigcirc$ | を | $\varepsilon$ | 3 | 5 | 7 |  |
| a | 1 | 8 | 8 | 1 | 6 |  |

o－2 1 type of vertical rotation， 720 degree rotation

| （4） | $\rightarrow$ | b |  | d | N－ | 6. | 10 |  | $\leftrightarrow$ | 万 | d |  | － | 6. | （1） | $\rightarrow$ | 6 | d |  | ： | 6. | － |  | $\rightarrow 1$ | 6 | d | c－ | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| － | $\cdots$ | ， |  | a． 1 | 10 | － | － | － | nt， | － | a． |  | $\bigcirc$ | － | － | a | $\bigcirc$ | a |  | in | － | $\checkmark$ |  | $\sim$ | \％ | $\cdots$ | 1n | － |
| $\infty$ | m | $t$ |  | ＋ | m | $\infty$ | $\infty$ |  | m | $\pm$ | ＋ | m | m | $\infty$ | $\infty$ | m | $t$ | 4 |  | m | $\infty$ | $\infty$ | 0 | m | $\pm$ | ＋ | $m$ | $\infty$ |
| $\infty$ | $\omega$ | ＋ |  | 7 | $\omega$ | $\infty$ | $\infty$ |  | $\omega$ | ＋ | 7 |  | $\omega$ | $\infty$ | $\infty$ | $\omega$ | F | 7 |  | $\omega$ | $\infty$ | $\infty$ |  | $\omega$ | $+$ | 7 | $\omega$ | $\infty$ |
| $\rightarrow$ | $v$ | － |  | or | r | － | － | V | $\checkmark$ | $\cdots$ | Or |  | $\checkmark$ | － | － | 0 | no | O |  | r | － | － | － | U | $\cdots$ | o | $\pi$ |  |
| ¢ | $\cdots$ | p |  | 9 | 5 | a－ | ， 0 | － | $\cdots$ | ¢ | 9 |  | 0 | co | o | $\cdots$ | $\stackrel{0}{ }$ | 9 |  | － | m | $\sigma$ | 0 | $\cdots$ | 10 | 9 | $\sim$ |  |
| （0） | －3 | 6 |  | a | N－ | 6. | ， 2 |  | － | 6 | d |  | cm | 6. | （0） | $\rightarrow$ | b | c） |  | E－ | 6 | 0 |  | $\rightarrow$ | b | d | t－ | 0. |
| － | $a$ | ，o | 0 | a | $1 \Omega$ | － | － | $n$ | $a$ | ，o | a |  | in | － | $\rightarrow$ | $n$ | ，o | o |  | in | － |  | $\rightarrow$ | $\cdots$ | ，o | a | in |  |
| $\infty$ | m | t |  | ＋ | m | $\infty$ | $\infty$ | c | $\cdots$ | t－ | ＋ |  | m | $\infty$ | $\infty$ | m | ＋ | － |  | m | $\infty$ | $\infty$ | $\infty$ | m | $\pm$ | ＋ | $m$ | $\infty$ |
| $\infty$ | $\omega$ | $\ddagger$ |  | 7 | $\omega$ | $\infty$ | $\infty$ | 0 | $\omega$ | ＋ | 7 |  | $\omega$ | $\infty$ | $\infty$ | $\omega$ | $\uparrow$ | 7 |  | $\omega$ | $\infty$ | $\infty$ | 0 | $\omega$ | $\ddagger$ | 7 | $\omega$ | $\infty$ |
| － | ज | － |  | or | $\pi$ | － | － | － | 0 | $\cdots$ | － | r | $\pi$ | － | － | ur | $\bigcirc$ | 0 |  | ro | － |  | － | $\cdots$ | \％ | $\cdots$ | $\pi$ |  |
| － | $\cdots$ | p |  | 9 | C－ | ro． | 0 | － | $\square$ | p | 9 |  | S－ | 8. | － 0 | $\rightarrow$ | － | $c$ |  | － | 0 | O | 0 | $\Rightarrow$ | $\stackrel{0}{0}$ | 9 | $5+$ | co |
| －0 | $\rightarrow$ | b |  | d | c－ | 6 | （2） | － | $\rightarrow$ | 6 | d |  | － | 6. | （Q） | $\rightarrow$ | 15 | c |  | ＊－ | 6 | Q | 2 | $\rightarrow$ | b | d | c | ¢． |
| － | $\sim$ | $\bigcirc$ |  | 0 | in | － | － | $\rightarrow$ | $\sim$ | ， | a |  | in | － | $\rightarrow$ | ni | 1. |  |  | is | － |  |  | $\sim$ | ． 10 | a | in |  |
| $\infty$ | m | 1 |  | $\pm$ | m | $\infty$ | $\infty$ | $\infty$ | m | $t$ | ＋ |  | m | $\infty$ | $\infty$ | m | $t$ | 7 |  | m | $\infty$ | $\infty$ | $\infty$ | m | $t$ | ＋ | $m$ | $\infty$ |
| $\infty$ | $\omega$ | $\pm$ |  | 7 | $\omega$ | $\infty$ | $\infty$ | $\infty$ | $\omega$ | $f$ | 7 |  | $\omega$ | $\infty$ | $\infty$ | $\omega$ | $+$ |  |  | $\omega$ | $\infty$ |  | $\infty$ | $\omega$ | $\dagger$ | 7 | $\omega$ | $\infty$ |
| － | ज | － |  | $\bigcirc$ | $\pi$ | － | － | － | $v$ | $\cdots$ | － |  | $\pi$ | － | － | vi | $\bigcirc$ |  |  | $\pi$ | － |  | － | 0 | $\cdots$ | $\sim$ | rv |  |
| －0 | 4 | 10 |  | 9 | ＜ | $\infty$ | － 0 | の－ | $\rightarrow$ | ¢ | 9 | 15 | 5 | 10 | － | $\cdots$ | p | $c$ |  | $5-$ | 2 |  | の | $\cdots$ | n | 4 | 5－ | co |
| （1） | $\rightarrow$ | 5 |  | a | 5－ | － 6 | －． 0 | 2 | $\rightarrow$ | $b$ | d | 15 | 5－ | ¢． | ， 0 | －s | is | $c$ |  | ＊ | $\omega$ |  |  | $\rightarrow$ | b | d | No | 6 |
| － | as | 10 |  | O | in | － | － | － | د | ．o | a | 1 | in | － | － | a | － |  |  | in | － |  | － | $\cdots$ | ， | a | 10 | － |
| $\infty$ | m | －t |  | ＋ | m | $\infty$ | $\infty$ | $\infty$ | m | $t$ | 7 |  | $m$ | $\infty$ | $\infty$ | $\cdots$ | $t$ |  |  | m | $\infty$ |  | 0 | $\cdots$ | t | $+$ | m | $\infty$ |
| $\infty$ | $\omega$ | F |  | 7 | $\omega$ | $\infty$ | $\infty$ | $\infty$ | $\omega$ | $f$ | 7 | 7 | $\omega$ | $\infty$ | $\infty$ | $\omega$ | F |  | 7 | $\omega$ | $\infty$ | $\infty$ | $\infty$ | $\omega$ | ¢ | 7 | $\omega$ | $\infty$ |
| － | 9 | 10 |  | a | IV | － | － | $-$ | 0 | $\cdots$ | 0 |  | ro | － | － | 0 | － |  |  | $\pi$ | － |  | － | v | $\cdots$ | o | rv | － |
| O | $\cdots$ | 1 | $\bigcirc$ | 9 | ＜－ | 0 | $0 \cdot 10$ | の | 4 | 1 | 9 | 15 | － | 0 | ． 01 | $\rightarrow$ | 1 |  | 4 | 5 | 1 | ， | 0 | $\cdots$ | \％ | 9 | － | 0 |


$1^{2} \times 16=n$
$1^{2} \times 16=16$


## "IKOSOLID"

The same number of real picture as " n " of real picture on Magic Squared Picture is shown on the surface of IKOSOLID. Because the order of arrangement is endless and infinite circulation, the nature of " n " is changed from finite to infinite. Therefore, it should be considered as "infinite multiplication of $n=n^{\infty}$ " The formula for real picture on the surface of IKOSOLID is:

$$
\mathrm{X}^{2} \times 16=\mathrm{n}^{\infty}
$$

( X is a vertical or horizontal number on real picture in 1 sixteenth of plane Magic Squared Picture.)

As the number of real picture on Magic Squared Picture is equivalent to " n " (multiplication is unnecessary) in this formula, the formula of Magic Squared Picture can "control" the number of real picture on the surface of IKOSOLID.
Therefore, the formula to work out the number of the real picture on Magic Squared Picture can be adapted for us to control when it is used as a catalyst on electronic base. On the surface of an IKOSOLID, the real picture would exist in millions, billions, trillions, quadrillions, or more in an infinite state. If "image base which is possible to change the number" is produced in the near future, simultaneous movement, time machine, beginning with dimensional penetration universal energy such as superconductivity at normal temperature, super-fluidity, can be visualized. An image base should be made as a next step of a base with vacuum tube and pc board in use currently. We believe that the number of base in real picture state can be freely controlled by integral formula for real picture number on Magic Squared Picture.

Photo 1 (IKOSOLID of pictorial art, points, numbers. Real pictures are symmetrical and existed in infinite circulation)


These 3 IKOSOLIDs have the same number of real pictures on the surfaces. The common formula is: $\mathrm{X}^{2} \times 16=\mathrm{n}^{\infty} 1^{2} \times 16=16^{\infty}$

Reference: Photos of IKOSOLIDs multi-combined and repeatedly combined at benzene combination are shown below. The IKOSOLIDs have both + and simultaneously on the surface. In the multi-combination and repeated-combination, it can be combined at the same pole and/or different pole. It should be considered that benzene combination used in molecule level and in chemical element level is transformed as solid.

Photo 2


KOEI ENDO
IKUYO ENDO
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# "Study of connecting point with three-dimensions and four-dimensions by pictorial art" 

## Part 3

## Moebius Strip and Magic Squared Picture, Klein's Bottle and IKOSOLID

August 2002
$\begin{array}{cc}\text { Koei } & \text { Endo } \\ \text { Ikuyo } & \text { Endo }\end{array}$

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# Study of connecting point with three-dimensions and four-dimensions by pictorial art 

Part 3<br>Moebius Strip and Magic Squared Picture, Klein's Bottle and IKOSOLID

## Introduction

IKOSOLID fundamentally exists in the four-dimensions and five-dimensions. It seemed paradoxical to prove this theory. It was proved in the part 2 of our paper, by combining two-dimensional origami linearly to express two relationships $\equiv ;$ follows:

1. "Relationship of the regular octahedron by Plato and IKOSOLID" in the four-dimensional geometry (topology and projective geometry) which is non-linear combination three-dimensionally
2. "Relationship of the Magic Squared Picture and IKOSOLID" in the four-dimensional integral calculus
Something made by a sheet of four-dimensional origami should be a five-dimensional solid as something made by a sheet of two-dimensional origami should be a three-dimensional solid. Then we thought that a certain solid to be shown five-dimensionally should be appeared three-dimensionally if a sheet of four-dimensional origami would be held two-dimensionally. That is our paradox and it led me and my wife, Ikuyo to create a mysterious solid, "IKOSOLID". It should be significant that Ikuyo successfully created IKOSOLID in the three-dimensional space through "our consciousness" which both of us have in common. Especially, I, as a professional painter, deeply appreciate what we have done. I have cooperated with Ikuyo to write our papers, part 1, 2, and 3.

We believe that painter's space recognition has been highly and intensely developed and we could have ability to express time and space by pictorial art in consequence of perspective drawing's research that successive famous and unknown painters had had. We would like to show and demonstrate the most advanced scientific discovery though we do not have much knowledge about science.

Research of time and space is a theme of most advanced art. Research of the dimensional penetration by pictorial art is also in progress. Painters have researched space and time by pictorial art, not scientifically and mathematically. Painters found four and five dimensional elements in the three-dimensional space expressed by perspective drawing. Paulo Uccello, Italian painter from 13 ${ }^{\text {th }}$ century to $14^{\text {th }}$ century, devoted his life to research of perspective drawing to express four and five dimensional elements in the two dimensional canvas. He roughly accomplished research of perspective drawing of showing rotating movement $\equiv$ real image and virtual image. Furthermore, Cezanne successfully adopted just three solids to express three-dimensionally 360 -degree rotating real image and virtual image. They are column, cone, and sphere. It was proved that a figure, which had created, by the painter and the architect should be a scientific molecule. The figure of "C60 $\mathrm{F} \equiv \mathrm{en}$ " which is lately popular is regarded as the same shape of figure created by painter, Ethsher, and architect, Fr In addition, Tra $\equiv \mathrm{u}$, Spanish painter died in 2001, painted the origin of universe and nature and has been appreciated by people who experienced outer space such as astronaut, Gagarin. We can say that he painted, by the painter's intuition, the fractal world of the nature and the universe. T Itsu said impressively that he had experienced something of his own creation about the origin of universe and nature in our natural environment.
Painters surly have ability to express by intuition the space beyond the dimension and microscopic world.
It is no wonder that this paper written under the mutual consciousness that Ikuyo and I have would be able to describe microcosm as macro, and high dimensions as two/three-dimensions. In this paper, we would like to discuss the similarity between Magic Squared Picture and IKOSOLID, and Moebius Strip and Klein's Bottle, which is four-dimensional solid of Moebius Strip. Moebius Strip and Klein's Bottle are essential in the topology.
We proposed in the part 2 of our paper that finiteness can control infiniteness. We would like to make it clearer herein.

August 2002

> Koei Endo
> Ikuyo Endo

## Chapter 1: Moebius Strip and Magic Squared Picture, Klein's Bottle and IKOSOLID

## 1. Relationship between Moebius Strip and Magic-Squared Picture, and Klein's Bottle and IKOSOLID

Moebius Strip... Twist an end of a strip of paper at 180 degrees (half turn) and glue the ends together. There are two kinds of twist, (regular) twist and reverse twist. It is a Torus $\mathrm{T}^{2}$ given (regular) twist, Klein's Bottle given reverse twist.


It is found that IKOSOLD is Moebius Strip itself, TorusT ${ }^{2}$ itself, and Klein's Bottle itself. It is the topic of this paper.

The process to form Moebius Strip from a strip of paper is very similar to make cross-combination of one half of Magic Squared Pictures (Figure 5, 6, 7). It is a complete two-dimensional diagram of IKOSOLID. It is plane diagram, side view, elevation that no professional architects and CAD have ever drawn. IKOSOLID exists topologically and projective geometrically. As perspective drawing (projective figure) is needed, it could not be described in plane diagram, side view, and elevation two-dimensionally. This two dimensional diagram is called as "K.I. $\oint \equiv b i n a t i o n " . ~ T h e ~ s o l i d ~ b y ~ " K . ~ I . ~ c o m b i n a t i o n " ~ i s ~$ cross-combination of one half of IKOSOLID. The solid combined is IKOSOLID. The way of cross-combination is called as "K. I. combination - 2 ".
The process to make the cross-combination of one half of Magic Squared Picture, "K. I. com 三ation", is similar the one to form the Moebius Strip. Moebius Strip is formed by twisting a strip of rectangular paper at 180 degrees and by gluing the ends together. It is easily "linearly combined". On the other hand, as "K. I. combination" is on the plane rectangle, it is impossible to be combined linearly. So, it is non-linearly combination. Only a solid 三hde by "K. I. combination-2 (cross-combination of one half of IKOSOLID)" can be combined linearly and turn to be a Moebius Strip.

The theory shall be discussed as follows.
The cross-combination of one half of Magic Squared Picture (K. I. combination) $\Rightarrow$ cross-combination of one half of IKOSOLID (K. I. combination - 2) $\Rightarrow$ IKOSOLID (solid as Moebius Strip, Torus $\mathrm{T}^{2}$, and Klein's Bottle.)

## 2. Formation process of Moebius Strip and the cross-combination of one half Magic Squared Pictures

(1) Formation process of Moebius Strip


Figure 1


Figure 1-2


Figure 1-3

In the Figure $1-3$, two strips of paper are overlapped at 180 degrees in the center. When two strips are twisted at 180 degrees and each edge is combined as $\mathrm{A}=\mathrm{A}^{\prime}, \mathrm{B}=\mathrm{B}^{\prime}$, C=C', D=D', they turn to be Moebius Strip.
If Moebius Strip by (regular) twist is covered with another Moebius Strip by (regular) twist, it turns to be Torus $\mathrm{T}^{2}$ (a solid which divides between space inside and space outside. Its shape is like a twisted float.)
If Moebius Strip by reverse twist is covered with another Moebius Strip by reverse twist, it turns to be Klein's Bottle (it is like a dimensional Moebius Strip which has both spaces inside and outside in common.)
(2) Formation process of the cross-combination of one half Magic Squared Pictures


Figure 2
Magic Squared Picture
(16 pieces of real picture as a sample)


Figure 3


Figure 4
When they are combined in the way of $A=A^{\prime}, B=B^{\prime}, C=C^{\prime}, D=D^{\prime}$, it can be like a Moebius Strip (refer to 3)

Two Figures, 1-3 and 4, are similar as figures before forming Moebius Strip.
Figure 1-3 is linear combination. Figure 4 is non-linear combination unless it is three-dimensional.
3. K. I. combination $=$ The cross-combination of one half magic-squared picture

Figure 5, 6, and 7 below called "K. I. combination" are detailed illustrations of Figure 4. It shows how two of one half Magic Squared Pictures are cross-combined. It describes the non-linear combination in which they are not to be combined in the way of $\mathrm{A}=\mathrm{A}^{\prime}$, $\mathrm{B}=\mathrm{B}$ ', and $\mathrm{C}=\mathrm{C}$ ' like Moebius Strip if they are not in the position of "K. I. combination-2" (Picture 1, 2, 3, and 4) (Photo 1).
"K.I. combination" is a "perfect plan of IKOSOLID". In addition, a real picture, which composes Magic Squared Picture, it would be a fundamental plan when IKOSOLID and Magic Squared Picture are scientifically adopted for the use of PC boards. It is indispensable two-dimensional figure in order to understand IKOSOLID on a plane.

By combining as $1=1^{\prime}, 2=2^{\prime}, 3=3^{\prime}, 4=4^{\prime}, 5=5^{\prime}, 6=6,7=7$, and $8=8^{\prime}$, two pieces of half IKOSOLID are made. Each edge of A, B, C, D, and A', B', C', D' are open, in these IKOSOLIDs. The cross-combination of one half IKOSOLID as described here is called as "K. I. combination-2" (Picture 1, 2, 3, and 4).
Furthermore, the figure in which each edge is combined as $A=A^{\prime}, B=B, C=C^{\prime}$, and $D=D^{\prime}$ is an IKOSOLID.


* When Side a and Side b are combined to form one half IKOSOLID, each side facing
each other is folded inside of two of one half IKOSOLID.

3'. The Magic Squared Picture is described as un-figured as follows in order to make the combination of "K. I. combination" more precise and clearer.


Side b


Figure 6


Figure 7'

- When Side a and Side b are combined to form one half IKOSOLID, each side facing each other is folded inside of two of one half IKOSOLID.


## 4. "K. I. combination-2" = the cross combination of one half IKOSOLID

 It is the figure just before Magic Squared Picture becomes Moebius Strip; the figure just before the plane/finite Magic Squared Picture transforms to the dimensional/infinite and endless (circulating) Magic Squared Picture, the figure just before the finite transforms to the infinite."K. I. combination-2" (= the cross combination of one ha
Side a IKOSOLID)
Becomes the inside wall of one Picture 1 (The opening part) Picture 2 (Back of the opening part) half real picture


Figure 8
Side b
Becomes the outside wall of one half real picture


Figure 9

Picture 3 (The opening part)


Picture 4 (back of the opening part)


When the edges $5=5,6=6,7=7$, and $8=8$, are combined, one half of IKOSOLID shown as Picture 3 and 4 is made.


Photo 1

* The Magic Squared Picture is described as un-figured in order to make the combination of Magic Squared Pictures on the wall of one half of IKOSOLID more precise and clearer. The Magic Squared Pictures are put all over IKOSOLID.


## 5. IKOSOLID possessing three elements of Moebius Strip, Torus $\mathrm{T}^{2}$, and Klein's Bottle

Moebius Strip has no surface or reverse side. In "K. I. combination-2", two pieces of one half IKOSOLID facing each other are cross-combined to be IKOSOLID. (A=A', $\mathrm{B}=\mathrm{B}, \mathrm{C}=\mathrm{C}$, and $\mathrm{D}=\mathrm{D}$ ' combination). It not only has a Moebius Strip's element, "having no surface or reverse side", but also has an element of Torus T ${ }^{2}$, "outs 三and inside walls are surface and reverse sides" = "inside and outside walls of Torus $T^{12}$ are distinguished from either surface or reverse side of the Magic Squared Picture put circularly and endlessly". We can also say that it has an element of Klein's Bottle, "having both inside and outside walls of Klein's Bottle in common" = "inside and outside walls have $50 \%$ surface and $50 \%$ reverse side of Magic Squared Picture and circulate infinitively and endlessly".
Considering that Magic Squared Picture consists of real picture which has no surface or reverse side, we come to the conclusion that the outside and inside walls of IKOSOLID should have the walls which have all of three elements of Moebius Strip, Torus $\mathrm{T}^{2}$, and Klein's Bottle.

## THEREFORE:

Moebius Strip $\rightarrow$ Cover a strip of Moebius Strip with a strip of Moebius Strip $\rightarrow$ Giving (regular) twist - Torus $\mathrm{T}^{2}$. Giving reverse twist - Klein's Bottle

In the "K. I. combination", two pieces of one half Magic Squared Picture cross-facing each other become one piece of half IKOSOLID. The opening part, A and A', B and B', C and C', and D and D' is opposite to each other and makes "K. I. combination-2". It becomes IKOSOLID by combining A with A', B with $\mathrm{B}^{\prime}, \mathrm{C}$ with $\mathrm{C}^{\prime}$, and D with $\mathrm{D}^{\prime}$.
"K. I. combination" surface and reverse side of Side a and Side b in the cross-combination of half Magic Squared Picture.
The nature of IKOSOLID depends on the conditions.
In the "K. I. combination-2", Side a is inside of half IKOSOLID and Side b is outside of half IKOSOLID. The relationship of "K. I. combination" and "K. I. combination-2" can decide the nature of inside and outside wall of IKOSOLID.
o-1 In the condition that both Side a and b are surfaces (called ""K. I. combination with the surface of Side a and Side b) $\rightarrow$ The inside wall of half IKOSOLID which has Side a as a wall is a surface (Side a) and outside is the reverse side. The inside
wall of half IKOSOLID which has Side b as a wall is the reverse and outside is the surface (Side b). It is called "K. I. combination with the surface of Side a and Side b". A solid made by these combinations is called " $50 \%$ of surface and $50 \%$ of reverse side of outside wall, $50 \%$ of surface and $50 \%$ of reverse side of inside wall of IKOSOLID".

The details of $\circ-1$ is as follows:


Surface


Reverse side

NOTE: Surface and reverse side on Magic Squared Picture is described as blank in order to make it clearer and easier.
(1) K.I. Combination with the surface of Side $a$ and Side $b \rightarrow$ K.I.-2 combination with the surface of a and $\mathrm{b} \rightarrow 50 \%$ of surface and $50 \%$ of reverse side of outside, $50 \%$ of surface of and $50 \%$ of reverse side of inside wall of IKOSOLID ... It should be considered that IKOSOLID can be in the position of Moebius Strip and Klein's Bottle as well if the penetration of real pictures (surface is equivalent to reverse side. Upper and lower • right and left are positioned symmetrically) existing mutually in the outside and inside of IKOSOLID is assumed.
a is the surface of inside.


Figure 10

Outside


Figure 12

Inside consists of $50 \%$ of surface and $50 \%$ of reverse side of Magic Squared Picture endlessly expanding.

Outside consists of $50 \%$ of surface and $50 \%$ of reverse side of Magic Squared Picture endlessly expanding.


Figure 12'
(2) K. I. combination with the reverse side of Side a and Side b $\rightarrow$ K. I.-2 combination with the reverse side of Side a and Side b $\rightarrow 50 \%$ of surface and $50 \%$ of reverse side of outside, $50 \%$ of surface and $50 \%$ of reverse side of inside wall of IKOSOLID ... It should be considered that IKOSOLID can be in the position of Moebius' Stripe and Klein's Bottle as well if the penetration of real pictures (surface is equivalent to reverse side. Upper and lower /right and left are positioned symmetrically) existing mutually in the outside and inside of IKOSOLID is assumed.

b is the reverse side of outside.

Figure 13


Figure 15"
(3) K. I. combination with the surface of Side a and the reverse side of Side b $\rightarrow$ K. I. combination 2 with the surface of Side a and the reverse side of Side $\mathrm{b} \rightarrow$ IKOSOLID consisting of the reverse side of outside and the surface of inside wall of IKOSOLID ... It should be considered as in the position of Torus $\mathrm{T}^{2}$, not of Moebius Strip. But it can be also in the position of Moebius Strip and Klein's Bottle if the penetration of real picture (surface is equivalent to reverse side. Upper and lower /right and left are positioned symmetrically) existing mutually in the outside and inside of IKOSOLID is assumed.



All over the outside consists of reverse side of Magic Squared Picture, endlessly expanding.

Inside wall
Figure 18
Outside

All over the inside wall consists of the surface of Magic Squared Picture, which is endlessly expanding.


Figure 18'
(4) K. I. combination with the reverse side of Side a and the surface of Side b $\rightarrow$ K. I. combination 2 with the reverse side of Side a and the surface of Side $\mathrm{b} \rightarrow$ IKOSOLID consisting of the surface of outside wall and the reverse side of inside wall ... It should be considered as in the position of Torus $\mathrm{T}^{2}$, not of Moebius Strip. But it can be also in the position of Moebius Strip and Klein's Bottle if the penetration of real picture (surface is equivalent to reverse side. Upper and lower /right and left are positioned symmetrically) existing mutually in the outside and inside of IKOSOLID is assumed.


Fi gure 19
a is the reverse side of inside.


Figure 20 outside.


All over the outside consists of surface of Magic Squared Picture endlessly expanding.
Figure 21
Inside


Figure 21’

It should be remarked that the real pictures in Magic Squared Picture are positioned symmetrically in the right and left and upper and lower. If the nature of real picture (penetration in the surface and reverse) can be used advantageously as it has, it can be considered that IKOSOLID can have the elements to be Moebius Strip, Klein's Bottle, and Torus $\mathrm{T}^{2}$ under the condition of (1), (2), (3), and (4). Moreover, it does not matter which side (surface or reverse side) is considered.
This paper is written on the basic theory that the interface of outside and inside should be penetrable by the idea of "the nature of real picture is PENETRATION OF SURFACE AND REVERSE SIDE = equivalence" when the dimensional Magic Squared Picture circulating infinitely and endlessly in the inside and outside of IKOSOLID consists of the real pictures in the plane Magic Squared Picture. In the Part 1 and 2 of our paper, the penetration of surface and reverse side on real picture is described as 720 degrees rotation.

# "Study of connecting point with three-dimensions and four-dimensions by pictorial art" 

## Part 4

Phase transformation of super-conductivity Perovskite structure is IKOSOLID
(Projected regular octahedron by Plato is a quarter of an IK OSOLID i.e. Four conjoined regular octahedra by Plato are an IK OSOLID)

J anuary 12, 2004
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